A Discrete Constitutive Model for Course Granular Soil

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Abstract

Rigid contact model for course-grained soil particles is established combined with the distribution of contact force and normal of contact force based on the research of contact characteristics of particles and ignoring the deformation of particle. The local constitutive model is acquired after analyzing the relationship between contact force of particles' and local stress in course-grained soil. Furthermore, a two dimensional discrete constitutive model is built up. It is proved through discrete constitutive model that the fabric will change during the deformation of course-grained soil which results the change of physical and mechanical characteristics. The change of fabric affects the macro mechanical responses characteristics of course-grained soil.

Keywords

Course-grained Soils; Discrete Granular Particle; Fabric; Constitutive Model

Introduction

Coarse-grained soil such as gravel soil, soil-rock aggregate mixture and rockfill is a typical granular media composed of discrete particles within two or three orders of magnitude in dimension. And it is widely applied in highway, railway, hydraulic dams and high embankment engineering. The physical and mechanical characteristics are quite different from traditional solid medium. Coarse-grained soil is a loose media in substance, and the contact between particles is discontinuous point-contact. From the paper of Luan Maotian, for coarse-grained soils and other loose granular media, the research of physical constitutive model of deformation and strength characteristics to overcome inherent limitation of mechanical methods of continuous medium with basis of micro mechanics is quite potential. Therefore, particle characteristic of coarse-grained soils, interaction and essential fabric of spatial uniform distribution between particles can be descripted objectively. Based on mathematical function of statistical distribution of the reflected fabric parameters, the relationship could be set up between inherent evolution of microstructure fabric parameters coarse-grained soils and macroscopic mechanics responses .

Granular media mechanics is assumed that coarse-grained soil is composed of solid particles contacted with each other, and interaction between particles obeys the laws of probability. Granular media mechanics is applied in studying the mechanical phenomena on the contact points of particles, and describes the phenomena according to the formula of mathematical statistics. Some mechanical models have been built by deferent research. However, those models can't interpret the relationship between fabric change and mechanical response of coarse-grained soils under load very well.

Rigid Particle Contact Model of Coarse-Grained Soil

There is a certain gradation within coarse-grained soil particles where large particles act as the skeleton and small particles fill in the void between the large particles. Both the interaction between particles and the filled surrounding medium and particles are belong to solid contact mechanics in terms of the mechanical characteristics. It is supposed that the number of coarse-grained soil particles is enormous amount, so the macro mechanical parameters have statistical significance. The uneven stress of microcosmic can be described by average stress. When the coarse-grained soil particle number is infinite, and coarse-grained soil is continuous in the certain space,. the summation in the calculation could be changed into integral.

The particle contact normal density distribution

function $E(\vec{n})$ is introduced, where $\vec{n}$ is the contact unit normal vector. The number of contact points within the $\vec{n} \to \vec{n}+d\vec{n}$ is $E(\vec{n})d\vec{n}$. Supposing $f_i(\vec{x}^\alpha,\vec{n})$ is the contact force component located at the contact point $\vec{x}^\alpha$ with normal $\vec{n}$. For a certain volume V of the coarse-grained soil, the total contact vectors is zero based on the conditions of static equilibrium.

$$\int f_i(\vec{n})E(\vec{n})d\vec{n} = 0 \qquad (i = x, y) \tag{1}$$

Where $f_i(\vec{n})$ is the average of contact force within $\vec{n}+d\vec{n}$.

The contact between particles is one of the most fundamental problems in particle mechanics model. Assumed that the coarse-grained soil particles are rigid, and the deformation of particles is ignored, so the contact of coarse-grained soil particles is elastic contact. Coarse-grained soil appears sliding or rolling possibly under load, so that two adjacent particles can be viewed as two rigid bodies connected by deformed springs on the contact point. Therefore, the deformation of particles is transformed into spring deformation under the contact force. Chose two similar particles of P, Q in the coarse-grained soil, it is shown in FIG. 1.

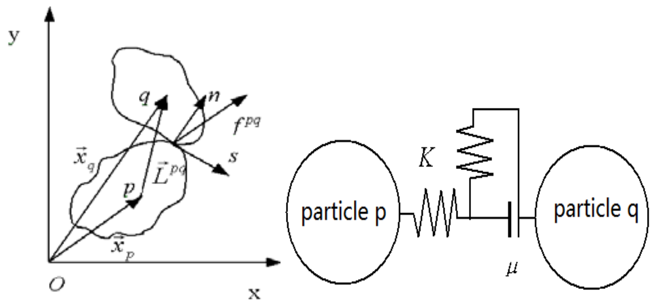

FIG 1. THE RIGID CONTACT MODEL OF COARSE-GRAINED SOIL

For particle P, the equilibrium equation can be established.

$$\begin{aligned}&\sum_{\alpha=1}^{m} f_i^{p\alpha}(\vec{x}^\alpha,\vec{n}')(x_j^{p\alpha}-x_j^{p})\\ &=\sum_{\alpha=1}^{m} f_i^{p\alpha}(\vec{x}^\alpha,\vec{n}')(x_i^{p\alpha}-x_i^{p})\end{aligned} \tag{2}$$

Where M is the contact number of particle p; $\vec{n}'$ is a unit branch vector, and $\vec{n}' = \dfrac{\vec{x}^p - \vec{x}^q}{\left|L^{pq}\right|}$, $\left|L^{pq}\right|$ is branch length.

Put the summation of all particles into integral form in course-grained soils.

$$\int f_i(\vec{n})L_j(\vec{n}')E(\vec{n})d\Omega = \int f_j(\vec{n})L_i(\vec{n}')E(\vec{n})d\Omega \tag{3}$$

Where $L_i(\vec{n}')$ is the average of contact force of the branch vector within $\vec{n}+d\vec{n}$.

Under the contact force, the strain tensor is $\varepsilon_{ij}(\vec{x}^\alpha)$, the displacement of contact point is $u_i(\vec{x}^\alpha)$. Ignoring rotation of particles, the equation is shown as below

$$u_i(\vec{x}^\alpha) = \varepsilon_{ij}(\vec{x}^\alpha)L_j(\vec{x}^\alpha,\vec{n}') \qquad (i, j = x, y) \tag{4}$$

The virtual work of contact force in the unit volume is

$$W = \frac{1}{2V}\sum f_i(\vec{x}^\alpha,\vec{n})\varepsilon_{ij}(\vec{x}^\alpha)L_j(\vec{x}^\alpha,\vec{n}') \tag{5}$$

Where Factor 2 represents each particle counted twice.

The virtual work for average stress is

$$W = \bar{\sigma}_{ij}\varepsilon_{ij}$$

From equation (2), (5), considering the symmetry of the stress tensor, the equation is obtained as follow.

$$\begin{aligned}\bar{\sigma}_{ij} = \frac{1}{4V}\sum_{\alpha=1}^{M}[&f_i(\vec{x}^\alpha,\vec{n})L_j(\vec{x}^\alpha,\vec{n}')\\ &+ f_j(\vec{x}^\alpha,\vec{n})L_i(\vec{x}^\alpha,\vec{n}')]\end{aligned} \tag{6}$$

The integral form is

$$\bar{\sigma}_{ij} = \frac{M}{4V}\int [f_i(\vec{n})L_j(\vec{n}) + f_j(\vec{n})L_i(\vec{n})]E(\vec{n})d\vec{n} \tag{7}$$

Equation (7) links the macroscopic average stress tensor and microscopic first-order tensor together. The coarse-grained soil particle size can be classified into l classes, and each class has an average particle size $\bar{d}_k$ with N_k particles. Assuming that all the coarse-grained soil particle size is $\bar{d}$, then

$$V_S = M\pi\bar{d}^2,\ \bar{d} = \frac{1}{2}\sqrt{\sum_{k=1}^{l}\bar{d}_k^{\,2}}$$

So

$$\begin{aligned}\bar{\sigma}_{ij} = &\frac{\bar{m}}{\pi(1+e)}\left(\sum_{k=1}^{l}\bar{d}_k^{\,2}\right)^{-1/2}\\ &\bullet\int_0^{2\pi}[f_i(\theta)n_j + f_j(\theta)n_i]E(\theta)d\theta\end{aligned} \tag{8}$$

Where $\bar{m}$ is the average coordination number; E is void ratio.

Assuming that the contact force is f_i (i=n, s; n, s as the local coordinate), then the relation between force and displacement on the contact point can be expressed as in incremental form.

$$\Delta f_i = D_{ij}\Delta U_j \tag{9}$$

Where D_{ij} is contact stiffness tensor.

In local coordinates n, s, D_{ij} can be expressed as

$$D_{ij} = D_n n_i n_j + D_S S_i S_j \tag{10}$$

Generally, the normal contact stiffness is a function of the normal contact force.

$$D_n = C_1 f_n^{\beta} \tag{11}$$

Where C, β are function associated with the coarse-grained soil characteristic, particle size and surface roughness.

The tangential stiffness is

$$D_s = C_2 D_n (1 - \frac{f_s}{f_n \tan\varphi_\mu})^{\eta}$$

Where C_2, η are coefficients related to the material itself; ϕ_μ is frictional angle between particles.

Two-dimensional Granular Material Constitutive Model

Coarse-grained soil particles are randomly stacked, so it can be described by fabric density distribution function. According to findings of Rothenburg and Bathurs, the density distribution fuction of coarse-grained soil particle fabric content could be described approximately by three fuction in the two-dimensional cases.

Contact normal vector

$$E(\theta) = \frac{1}{2\pi}(1 + a_1(\cos\theta - \theta_1)] \tag{12}$$

Normal contact force

$$f_n(\theta) = f_0[1 + a_2 \cos 2(\theta - \theta_2)] \tag{13}$$

Tangential contact force

$$f_t(\theta) = -f_0 a_3 \sin 2(\theta - \theta_3) \tag{14}$$

Where the minus is Negative tangential contact force to rotate counter clock wise positive; $\alpha_1, \alpha_2, \alpha_3$ are coefficients reflecting the degree of anisotropy; $\theta_1, \theta_2, \theta_3$ denote fabric shaft angle, maximum normal contact force, average contact angle of maximum tangential contact force respectively.

f_0 is average tangential contact force.

$f_0 = \int_0^{2\pi} f_n(\theta) d\theta$

By substitution formula (12-14) into the formula (8)

$$\begin{aligned}\bar{\sigma}_{11} = \frac{\bar{m} f_0}{(1+e)\pi}\left(\sum_{k=1}^{l} \bar{d}_k^2\right)^{-1/2} [1 + \frac{1}{2}(a_1 \cos 2\theta_1 + \\ a_2 \cos 2\theta_2 + a_3 \cos 2\theta_3) + \frac{a_1 a_2}{2}(\cos 2\theta_1 \cos 2\theta_2 \\ + \sin 2\theta_1 \sin 2\theta_2)]\end{aligned} \tag{15}$$

$$\begin{aligned}\bar{\sigma}_{22} = \frac{\bar{m} f_0}{(1+e)\pi}\left(\sum_{k=1}^{l} \bar{d}_k^2\right)^{-1/2} [1 - \frac{1}{2}(a_1 \cos 2\theta_1 \\ + a_2 \cos 2\theta_2 + a_3 \cos 2\theta_3) + \frac{a_1 a_2}{2}(\cos 2\theta_1 \cos 2\theta_2 \\ + \sin 2\theta_1 \sin 2\theta_2)]\end{aligned} \tag{16}$$

$$\begin{aligned}\sigma_{12} = \frac{\bar{m} f_0}{(1+e)\pi}\left(\sum_{k=1}^{l} \bar{d}_k^2\right)^{-1/2} (a_1 \sin 2\theta_1 \\ + a_2 \sin 2\theta_2 + a_3 \sin 2\theta_3)\end{aligned} \tag{17}$$

Stress is associated with void ratio, the number of contact points and fabric contents. If the relationship between contact force and contact displacement is linear, the formula is obtained as follow by Hooke's law.

$$\Delta\sigma = E\Delta\varepsilon$$

The tangential stiffness

$$D_s = \lambda D_n$$

Where λ is a constant, generally $\lambda = 0.1 - 1.0$.

The relationship between strain and stress of coarse-grained soil can't be derived directly, and the strain should be linked to contact force. Coarse-grained soil relationships between stress and strain can be derived when the local constitutive relation is confirmed.

The contact stiffness tensor is

$$D_{ij} = D_n n_i n_j + D_s s_i s_j \qquad (i, j = 1, 2) \tag{18}$$

Local contact constitutive relation of incremental form

$$\Delta f_i = D_{ij} \Delta u_j \tag{19}$$

Where Δf_i is incremental contact force on the contact point; Δu_j is incremental displacement on the contact point.

For the coarse-grained soil, the number of particles is homogeneous on the large scale, and its displacement is linear distribution without forming shear zone.

$$\Delta u_j = l_i \Delta\varepsilon_{ij} = l(\vec{x}^{\alpha}) n_i \Delta\varepsilon_{ij} \tag{20}$$

Where $\Delta\varepsilon_{ij}$ is strain increment.

The relation between contact stress increment and strain increment is

$$\Delta f_i(\vec{x}^{\alpha},\vec{n}) = l(\vec{x}^{\alpha})(D_n n_i n_j + D_s s_i s_j) n_k \Delta\varepsilon_{kj} \quad (i, j = 1,2) \tag{21}$$

By substitution formula(21)into formula(20), the incremental formula of force and strain is

$$\Delta\sigma_{ij} = A_{ijkl}\Delta\varepsilon_{ij} \quad (i, j = 1,2) \tag{22}$$

Where

$$A_{ijkl} = \frac{2\bar{m}}{(1+e)\pi}\int_0^{2\pi}(n_i n_j n_k n_l + B_{ijkl}D_s)E(\theta)d\theta \tag{23}$$

$$B_{ijkl} = (n_i s_j n_k s_l + n_j s_i n_k s_l + n_i s_j n_l s_k + n_j s_i n_l s_k)/4$$

Density function is

$$E(\vec{n}) = 1/2\pi \tag{24}$$

Substitute formula (24) into formula (23), and integral is

$$\begin{bmatrix}\Delta\sigma_{xx}\\ \Delta\sigma_{yy}\\ \Delta\sigma_{xy}\end{bmatrix} = \frac{\bar{m}}{4(1+e)\pi} \bullet \begin{bmatrix}3D_n+D_s & D_n-D_s & 0\\ D_n-D_s & 3D_n+D_s & 0\\ 0 & 0 & D_n+D_s\end{bmatrix}\begin{bmatrix}\Delta\varepsilon_{xx}\\ \Delta\varepsilon_{yy}\\ \Delta\gamma_{xy}\end{bmatrix} \tag{25}$$

The bulk modulus, shear modulus and Poisson's ratio are respectively

$$K = \frac{\bar{m}D_n}{2(1+e)\pi}$$

$$G = \frac{\bar{m}(D_n+D_s)}{4(1+e)\pi}$$

$$\upsilon = \frac{D_n-D_s}{3D_n-D_s}$$

It is proved that the bulk modulus and shear modulus of Coarse-grained soil is related to pore ratio and coordination number.

Conclusions

The physical and mechanical properties of coarse-grained soil are closely related to particle spatial stacking mode, void size and the distribution of void space. The relationship between average force and contact point is set up by analyzing the force between particles and normal direction of contact based on the fabric characteristics of coarse-grained soil. Due to fabric change during the process of deformation, relation between force and strain is nonlinear as same as relation between force and contact force, the relationship between stress and contact force is affected by the fabric. The mechanical response of coarse-grained soil in the large scale proved that the changes of fabric contents play an important role on the deformation characteristics of materials.

The rigid contact model of coarse-grained soil is established based on the microstructure of coarse-grained soil particles. Local constitutive relation of coarse-grained soil is acquired by the connection between contact force of local stress between particles. Based on the local constitutive relation of coarse-grained soil, the constitutive relation of two-dimensional granular of coarse-grained soil has been set up. These studies provide foundations for the further researches.

The application of basic theories of discrete mechanics and particle mechanics is a promising method to study stress-strain characteristics of coarse-grained soil. The two-dimensional constitutive relation has been discussed in this paper. For topics related to three-dimensional which is quite similar to the two-dimensional approaches. And the contact force of coarse-grained soil particles and contiguous normal distribution need further researches.

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